Generalized Bloch equations for a strongly driven tunneling system

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Using the *Robertson projection operator formalism*, we derive generalized Bloch equations which describe the dynamics of a biased two-level tunneling system strongly driven by an external field and weakly coupled to a super-Ohmic heat bath. The generalized Bloch equations constitute a set of coupled nonlinear integro-differential equations. With their help we investigate the influence of phonons on the phenomenon of dynamical localization. [S1063-651X(97)05002-2]

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I. INTRODUCTION

The last decade has witnessed detailed investigations of the problem of macroscopic quantum coherence [1,2]. The generic model is a two-level tunneling system (TLS) coupled to a harmonic heat bath and an external driving field. The tunneling units may represent, e.g., small groups of atoms or molecules in structurally disordered solids, such as in the study of low-temperature properties of glasses [3]; electrons in semiconductor quantum wells [4] or in the electron-transfer dynamics in chemical reactions [5]; or magnetic flux quanta at Josephson junctions or superconducting quantum interference devices (SQUIDs) [6]. The basic mechanism by which quantum coherence is destroyed is the coupling of the quantum system to the dissipative environment. Various approaches using second-order perturbation theory, variational polaron theory [7], mode-coupling theories [8] or, most popular, the noninteracting-blip approximation (NIBA) [1,2] have been used to discuss the decoherence effect of the bath.

One key issue is whether by careful tuning of the driving field the effect of dissipation may be reduced and quantum coherence be restored on the macroscopic level. This idea of controlling tunneling and relaxation has become especially popular since the discovery, by Grossmann, Hänggi, and coworkers [9], of the effect of *coherent destruction of tunneling*. In a numerical treatment of an isolated bistable quartic potential driven by a periodic force—but not coupled to a heat bath—they found complete suppression of tunneling: at certain ratios of the field amplitude and the driving frequency the two lowest Floquet eigenstates (quasienergies) of the driven system become degenerate, thereby preventing the system from performing coherent oscillations between the left and right well. As a result, a particle initially localized in one well will remain there forever. This so-called *dynamical localization* effect has also been described in a two-state model [10–12], and has since been the subject of several further investigations [13–22].

Recent studies have focused on the question of to what extent the dynamical localization of a TLS will be affected by the coupling to a heat bath. Physical intuition seems to suggest that in order for dynamical localization to occur, a wave packet initially localized in one well must have a well-defined phase; only then can one expect that a mismatch between the tunneling motion and the driving field may prevent a particle from escaping into the other well. Since phonons destroy this phase coherence—i.e., there is a finite phase memory time τ_2—dynamical localization should always be softened through the coupling to a heat bath.

Recently Grifoni *et al.* [13] gave a systematic approach to the transient and steady-state dynamics of a driven dissipative TLS. They considered Ohmic and frequency-dependent damping in the framework of the NIBA for the stochastic force, but without any approximation for the driving force. In Ref. [14] they extended their approach beyond the NIBA. Dittrich *et al.* [15] investigated numerically the effect of driving and dissipation on the coherent tunneling motion of a symmetric bistable system for weak Ohmic damping. Dakhnovskii [16,17] employed small-polaron theory to address the same issue in a two-state approximation within the NIBA. He concluded for the case of a symmetric TLS with super-Ohmic damping that, in contrast to intuition, the localization transition remains stable against disturbance by the bath as long as the driving frequency is larger than the relaxation energy (polaron-binding energy) of the lattice. On these grounds he predicted the existence of a slow mode oscillation near the transition [16]. His conclusions essentially rely on the fact that near the localization transition the rate of phase loss—which is due to the bath—decreases faster than the tunneling coherence.

So far most investigations have made use of the NIBA. Yet the NIBA breaks down at nonzero bias, weak dissipation, and low temperatures. It is the latter regime, so far poorly understood, which we shall discuss in this paper. Weak TLS-phonon coupling is the limit which is relevant for, e.g., the description of tunneling centers in dielectric solids. We will work within the two-state picture and assume a super-Ohmic spectral density for the phonons; and we will treat the TLS-phonon coupling in Born approximation [23]. (There are in fact general arguments why under the above assumptions it should always be justified to treat the dissipation perturbatively [1].) The external field, on the other hand, will be treated to any order. We are then able to derive non-

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linear and non-Markovian *generalized Bloch equations* (GBE). These in turn permit us to discuss the influence of weak dissipation on the localization transition for both the symmetric and the biased case, and for arbitrarily strong driving and any temperature. In the high-temperature and high-frequency limit there are analytical solutions, which we will compare with those obtained from the NIBA.

The paper is organized as follows. In Sec. II we give a short introduction to the Robertson formalism and apply it to driven TLS dynamics. In Sec. III we derive the GBE. The linear response regime is discussed as a special case in Sec. IV. In Sec. V we address the influence of weak dissipation on the dynamical localization transition, before we close (in Sec. VI) with a brief summary.

II. ROBERTSON FORMALISM

We will derive a non-Markovian generalization of the Bloch equations with the help of the so-called Robertson formalism [24]. The Robertson formalism constitutes a particular variant of the well-known projection technique [25], one that is tailored to describing the evolution of selected observables even far from equilibrium.

When studying the dynamics of a macroscopic quantum system, one is typically confronted with the problem of determining the evolution of only a small set of selected (''relevant'') observables $\{G_a\}$. Let the (generally time-dependent) Hamiltonian be denoted by $H(t)$. Then the equation of motion for the expectation values

$$g_a(t)\equiv\langle G_a\rangle_{\rho(t)}:=\mathrm{tr}[\rho(t)G_a] \tag{2.1}$$

reads

$$\dot{g}_a(t)=i\langle\mathcal{L}(t)G_a\rangle_{\rho(t)}, \tag{2.2}$$

with $\rho(t)$ being the statistical operator and $\mathcal{L}(t)$ the Liouvillian

$$\mathcal{L}(t):=\hbar^{-1}[H(t),*]. \tag{2.3}$$

Associated with the expectation values $\{g_a(t)\}$ is, at each time t, a generalized canonical state

$$\rho_{\mathrm{rel}}(t):=Z(t)^{-1}\exp\left(-\sum_a l^a(t)G_a\right), \tag{2.4}$$

called the *relevant part of the statistical operator*, with partition function

$$Z(t):=\mathrm{tr}\,\exp\left(-\sum_a l^a(t)G_a\right) \tag{2.5}$$

and the Lagrange parameters $\{l^a(t)\}$ adjusted such as to yield the correct expectation values (2.1) of the relevant observables. The difference

$$\rho_{\mathrm{irr}}(t):=\rho(t)-\rho_{\mathrm{rel}}(t) \tag{2.6}$$

is then the *irrelevant part* of the state.

Like all variants of the projection technique, the Robertson formalism is based on a clever insertion of projection operators into the equation of motion (2.2). Here the projection operator $\mathcal{P}(t)$ is chosen such that it projects arbitrary vectors in Liouville space onto the subspace spanned by the unit operator and by the relevant observables $\{G_a\}$, and that this projection is orthogonal with respect to the (time-dependent) scalar product

$$\langle A;B\rangle^{(t)}:=\int_0^1 d\mu\ \mathrm{tr}[\rho_{\mathrm{rel}}(t)^{\mu}A^{\dagger}\rho_{\mathrm{rel}}(t)^{1-\mu}B]. \tag{2.7}$$

Since this scalar product varies in time, the projection operator, too, is time dependent; so is its complement $\mathcal{Q}(t):=1-\mathcal{P}(t)$. The projection operator is known as the *Kawasaki-Gunton projector* [26].

Let $\mathcal{T}(t',t)$ be the evolution operator defined by

$$\frac{\partial}{\partial t'}\mathcal{T}(t',t)=-i\ \mathcal{Q}(t')\mathcal{L}(t')\mathcal{Q}(t')\mathcal{T}(t',t), \tag{2.8}$$

with initial condition $\mathcal{T}(t,t)=1$. As $\mathcal{Q}$ projects out the irrelevant component of an observable, this operator may be pictured as describing the evolution of the *irrelevant* degrees of freedom of the system. The equation of motion for the expectation values of the relevant observables can then be cast into the form

$$\dot{g}_a(t)=i\langle\mathcal{L}(t)G_a\rangle_{\mathrm{rel}(t)}+\int_0^t dt'\sum_c\ \langle\mathcal{Q}(t')\mathcal{L}(t')G_c;\mathcal{T}(t',t)\mathcal{Q}(t)\mathcal{L}(t)G_a\rangle^{(t')}l^c(t')+i\langle\mathcal{T}(0,t)\mathcal{Q}(t)\mathcal{L}(t)G_a\rangle_{\mathrm{irr}(0)}, \tag{2.9}$$

which is known as the *Robertson equation* [24]. Here $\langle\ \rangle_{\mathrm{rel}}$ and $\langle\ \rangle_{\mathrm{irr}}$ denote expectation values evaluated in the relevant and irrelevant part of the state, respectively. A major simplification occurs if, as is often the case, the initial macrostate can be characterized completely by the expectation values $\{\langle G_a\rangle(0)\}$ of the relevant observables and hence has the generalized canonical form (2.4). In this case the initial state has no irrelevant component, which in turn implies that the third (''residual force'') term vanishes.

The Robertson equation embodies a closed system of nonlinear coupled integro-differential equations for the expectation values $\{g_a(t)\}$. These coupled equations are nonlocal in time: future expectation values of the relevant observables are predicted not just on the basis of their present values, but on their entire history. In essence, the irrelevant degrees of freedom have been eliminated, but this elimination must be paid for by nonlinear and non-Markovian features of the projected equation of motion; the complexity of the evolution has been mapped onto nonlinearities and a nonlocal behavior in time.

We now determine the Robertson equation for a driven TLS. The underlying microscopic dynamics of the TLS is determined by a time-dependent Hamiltonian

$$H(t)=H_S(t)+H_{SB}+H_B. \tag{2.10}$$

Here the system Hamiltonian is given by

$$H_S(t)=-\tfrac{1}{2}\vec{\sigma}\cdot\vec{h}(t), \tag{2.11}$$

where $\vec{h}(t)=\hbar(\Delta_0,0,-\Delta-f(t))$ includes the tunneling frequency Δ_0, the bias Δ, and the time-dependent driving field

$$f(t)=2\Omega\cos(\omega_L t), \tag{2.12}$$

with Rabi frequency Ω and driving frequency ω_L. If we denote the state of a particle localized in the left (right) well by $|L\rangle$ ($|R\rangle$) then the Pauli matrix $\sigma_z=|L\rangle\langle L|-|R\rangle\langle R|$ represents its coordinate, while $\sigma_x=|L\rangle\langle R|+|R\rangle\langle L|$ represents its tunneling motion. With this convention $\Delta>0$ means that the left well has a higher potential energy than the right well. The bath Hamiltonian $H_B=\Sigma_k\hbar\omega_k b_k^\dagger b_k$, with bosonic creation and annihilation operators satisfying $[b_i,b_j^\dagger]=\delta_{ij}$, describes the dynamics of an ensemble of harmonic oscillators (phonons). Their spectral density is taken to be super-Ohmic,

$$J(\omega)=\frac{2}{\hbar^2}\sum_j c_j^2\delta(\omega-\omega_j)=U\omega^3\exp(-\omega/\omega_c),\quad \omega\geqslant 0 \tag{2.13}$$

up to some cutoff ω_c. Finally, the system-bath coupling is assumed to be bilinear in the tunneling and the phonon coordinate,

$$H_{SB}=\sigma_z\sum_j c_j(b_j+b_j^\dagger)=:\sigma_z e. \tag{2.14}$$

Relevant observables are

$$\{G_a\}\rightarrow\{\sigma_\alpha,(H_{SB}+H_B)\}, \tag{2.15}$$

with expectation values

$$\{g_a(t)\}\rightarrow\{p_\alpha(t),E_B(t)\} \tag{2.16}$$

and associated Lagrange parameters

$$\{l^a(t)\}\rightarrow\{-\beta\lambda_\alpha(t)/2,\beta\}. \tag{2.17}$$

Here $p_\alpha(t)$ denotes the time-dependent polarization of the TLS, and $E_B(t)$ the internal energy of the bath. We assume the heat bath to be large enough so that its inverse temperature $\beta=1/k_BT$ does not vary in time. The relevant part of the statistical operator reads

$$\begin{aligned}\rho_{\rm rel}(t)&=Z^{-1}\exp\{-\beta[H_B+H_{SB}-\tfrac{1}{2}\vec{\lambda}(t)\cdot\vec{\sigma}]\}\\&=Z^{-1}\exp(-\beta\{H(t)+\tfrac{1}{2}[\vec{h}(t)-\vec{\lambda}(t)]\cdot\vec{\sigma}\}).\end{aligned} \tag{2.18}$$

Provided the (full) initial state $\rho(0)$ can be written in this generalized canonical form, i.e., provided $\rho(0)=\rho_{\rm rel}(0)$, then the residual force term in the Robertson equation vanishes. The class of initial states in the generalized canonical form includes the common case of a particle initially held at the site $|L\rangle$, $p_z(0)=1$, and coupled to a bath in thermal equilibrium,

$$\rho(0)\propto\pi_L\exp\{-\beta[H(0)+\tfrac{1}{2}\hbar\Delta_0\sigma_x]\}\pi_L. \tag{2.19}$$

Here, $\pi_L=\frac{1}{2}(\mathbf{1}+\sigma_z)$ denotes the projector on the state $|L\rangle$. This initial condition has the generalized canonical form (2.4) with Lagrange parameters $\vec{\lambda}(0)=(0,0,\infty)$. Clearly, the fact that the initial state has the generalized canonical form does not imply that the total system is in equilibrium; rather, it means that the initial state can be completely characterized by the expectation values $\{p_\alpha(0),E_B(0)\}$ of the relevant observables. The Robertson equation then translates into the generalized Bloch equations

$$\begin{aligned}\dot{p}_\alpha(t)=\hbar^{-1}\{\vec{p}(t)\times[\vec{h}(t)-\vec{\lambda}(t)]\}_\alpha-\int_0^t dt'\sum_\beta K_{\alpha\beta}(t,t')\\ \times[\lambda_\beta(t')-h_\beta(t')],\end{aligned} \tag{2.20}$$

with memory kernel

$$K_{\alpha\beta}(t,t'):=\frac{\beta}{2}\langle\mathcal{Q}(t')\mathcal{L}_{SB}\sigma_\beta;\mathcal{T}(t',t)\mathcal{Q}(t)\mathcal{L}_{SB}\sigma_\alpha\rangle^{(t')}. \tag{2.21}$$

The GBE are non-Markovian: our formalism allows for the inclusion of arbitrarily large memory effects. Indeed, such non-Markovian effects will prove essential for the determination of the correct relaxation rates.

III. DRIVEN DYNAMICS AT WEAK DISSIPATION

For simplicity, we will from now on set $\hbar=k_B=1$. In order to evaluate the memory kernel we assume weak coupling between the TLS and the bath, and hence do lowest-(i.e., second-) order perturbation theory in H_{SB} (Born approximation). To this order we can omit the complement projectors $\mathcal{Q}$, replace the evolution operator $\mathcal{T}$ with

$$\mathcal{T}(t',t)\;\rightarrow\;\mathcal{U}_S(t',t)\otimes\mathcal{U}_B(t',t), \tag{3.1}$$

where $\mathcal{T}$, $\mathcal{U}_S$, $\mathcal{U}_B$ are the evolution operators associated with $\mathcal{QLQ}$, $\mathcal{L}_S$, and $\mathcal{L}_B$, respectively, and evaluate the scalar product (2.7) in the state

$$\begin{aligned}\rho_{\rm rel}^{(0)}(t)&=\rho_B\otimes\rho_S[\vec{\lambda}(t)]:\\&=\frac{1}{Z_B}\exp(-\beta H_B)\otimes\frac{1}{Z_S}\exp[\tfrac{1}{2}\beta\vec{\lambda}(t)\cdot\vec{\sigma}]\end{aligned} \tag{3.2}$$

rather than in $\rho_{\rm rel}(t)$. We thus obtain

$$K_{\alpha\beta}(t,t')=g(t-t')\Xi_{\alpha\beta}(t,t'), \tag{3.3}$$

with the bath relaxation function

$$g(t-t'):=2\beta\langle e;\mathcal{U}_B(t',t)e\rangle_B=2\int_0^\infty d\omega\,\frac{J(\omega)}{\omega}\cos[\omega(t-t')] \tag{3.4}$$

and spin relaxation function

$$\Xi_{\alpha\beta}(t,t'):=\tfrac{1}{4}\langle[\sigma_z,\sigma_\beta];\mathcal{U}_S(t',t)[\sigma_z,\sigma_\alpha]\rangle_\lambda^{(t')}. \quad (3.5)$$

Here $\langle\ \rangle_\lambda^{(t')}$ and $\langle\ \rangle_B$ denote scalar products evaluated in the states $\rho_S[\vec{\lambda}(t')]$ and ρ_B, respectively. As the spectral density $J(\omega)$ has a characteristic width ω_c, the bath relaxation function $g(\tau)$—which is essentially its Fourier transform—decays on a typical scale (''memory time'') $\tau_{\mathrm{mem}}\sim 1/\omega_c$. For super-Ohmic damping this memory time is the only characteristic time scale of the bath, independent of the temperature. This distinguishes the super-Ohmic from the Ohmic case where there exists a second characteristic scale of order $(KT)^{-1}$, where K is the Kondo parameter.

Using now, in Born approximation,

$$\vec{p}(t')=\chi(t')\vec{\lambda}(t'), \quad (3.6)$$

with

$$\chi(t'):=\frac{1}{|\vec{\lambda}(t')|}\tanh\frac{\beta|\vec{\lambda}(t')|}{2}, \quad (3.7)$$

and defining the instantaneous equilibrium polarization

$$\langle\vec{\sigma}\rangle_{h(t')}:=\chi(t')\vec{h}(t'), \quad (3.8)$$

the GBE can be cast into the compact form

$$\dot{p}_\alpha(t)=[\vec{p}(t)\times\vec{h}(t)]_\alpha-\int_0^t dt'\ g(t-t')\sum_\beta\ \Xi_{\alpha\beta}(t,t')\times\chi^{-1}(t')[p_\beta(t')-\langle\sigma_\beta\rangle_{h(t')}]. \quad (3.9)$$

This approximate equation of motion, valid at weak dissipation, is still non-Markovian, and still nonlinear in $\vec{p}$. Far from equilibrium the nonlinearities may become significant and lead to a nonexponential relaxation of the polarization vector. Also the external driving field enters nonlinearly, through the time-evolution operator in the spin relaxation function.

IV. LINEAR RESPONSE REGIME

To test our generalized Bloch equations we show that in the linear response regime they yield results consistent with earlier calculations. We assume that the dynamics take place in the linear regime, i.e., that at all times the polarization be close to its instantaneous equilibrium, $\vec{p}\approx\langle\vec{\sigma}\rangle$. In this regime we may evaluate both $\Xi(t,t')$ and $\chi^{-1}(t')$ in the instantaneous equilibrium state $\rho_S[\vec{h}(t')]$, rather than in the state $\rho_S[\vec{\lambda}(t')]$, thus rendering the GBE linear in $\vec{p}$. Approximating the spin relaxation function by

$$\Xi^{(0)}(t,t')=\begin{pmatrix}\cos(\epsilon_0\tau) & -\overline{u_0}\sin(\epsilon_0\tau) & 0\\ \overline{u_0}\sin(\epsilon_0\tau) & u_0^2+\overline{u_0^2}\cos(\epsilon_0\tau) & 0\\ 0 & 0 & 0\end{pmatrix}, \quad (4.1)$$

with $\tau:=t-t'$, $\epsilon_0:=\sqrt{\Delta^2+\Delta_0^2}$, $u_0:=\Delta_0/\epsilon_0$, and $\overline{u_0}:=\Delta/\epsilon_0$, and expanding the susceptibility in a Fourier series

$$\chi^{-1}(t)=\sum_{m=-\infty}^{\infty}\chi_m^{-1}\ e^{-im\omega_L t}, \quad (4.2)$$

the GBE can be solved by Laplace transformation. Keeping terms up to linear order in Ω one finds, after tedious but straightforward calculations,

$$p_z(t)=p_z^{(0)}(t)+p_z^{(1)}(t), \quad (4.3)$$

where the first term describes the transient dynamics, while the second term describes the steady-state dynamics. The transient term is given by the standard second-order result for a static bias [27],

$$p_z^{(0)}(t)=u_0^2e^{-t/\tau_2^{(0)}}\cos(\epsilon_0 t-\varphi)/\cos\varphi+(\overline{u_0^2}-p_z^{\mathrm{eq}})e^{-t/\tau_1^{(0)}}+p_z^{\mathrm{eq}}, \quad (4.4)$$

with $p_z^{\mathrm{eq}}=-\overline{u_0}\tanh(\beta\epsilon_0/2)$ and $\cot\varphi=\epsilon_0\tau_2^{(0)}$, and relaxation rates

$$(\tau_2^{(0)})^{-1}=(2\tau_1^{(0)})^{-1}=\tfrac{1}{2}u_0^2\pi U\epsilon_0^3\coth(\beta\epsilon_0/2). \quad (4.5)$$

The steady-state term gives the linear response of the TLS to the driving field. In the asymptotic regime where all transients have decayed, the polarization oscillates coherently with the driving frequency ω_L; in the linear regime there is no generation of higher harmonics. Defining the dynamical susceptibility $\chi(\omega_L)$ via the relation

$$p_z^{(1)}(t)=-2\Omega[\chi(\omega_L)e^{-i\omega_L t}+\chi(-\omega_L)e^{i\omega_L t}] \quad (4.6)$$

we find for resonant driving ($\omega_L\approx\pm\epsilon_0$)

$$\chi_{\mathrm{res}}(\omega_L)=\frac{u_0^2}{4}\tanh(\beta\epsilon_0/2)\sum_\pm\ \frac{\pm i\tau_2^{(0)}}{1-i(\omega_L\mp\epsilon_0)\tau_2^{(0)}} \quad (4.7)$$

and for low-frequency driving ($\omega_L\ll\epsilon_0$)

$$\chi_{\mathrm{rel}}(\omega_L)-\chi_\infty=\frac{\beta/4}{\cosh^2(\beta\epsilon_0/2)}\frac{\overline{u_0^2}}{1-i\omega_L\tau_1^{(0)}}, \quad (4.8)$$

where $\chi_\infty=(u_0^2/4\epsilon_0)\tanh(\beta\epsilon_0/2)$ denotes the instantaneous response. In the derivation we have made use of $\chi_0\chi_{\pm1}^{-1}=(\Omega\overline{u_0}/\epsilon_0)\{1-[\beta\epsilon_0\coth(\beta\epsilon_0/2)]/[2\cosh^2(\beta\epsilon_0/2)]\}$. These results are well known from the literature, see, e.g., Ref. [28].

V. EFFECT OF WEAK DISSIPATION ON DYNAMICAL LOCALIZATION

We return to the general case of dynamics arbitrarily far from equilibrium. We factor out that part of the dynamics which is due to the driving field alone, by defining new polarization vectors

$$\tilde{p}_\alpha(t):=\sum_\beta\ R_{\alpha\beta}(t)p_\beta(t), \quad (5.1)$$

$$\langle\tilde{\sigma}_\alpha\rangle_{h(t)}:=\sum_\beta\ R_{\alpha\beta}(t)\langle\sigma_\beta\rangle_{h(t)} \quad (5.2)$$

via an orthogonal rotation matrix

$$R(t):=\begin{pmatrix} \cos F(t) & \sin F(t) & 0 \\ -\sin F(t) & \cos F(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5.3}$$

with

$$F(t):=\int_0^t f(s)ds=\frac{2\Omega}{\omega_L}\sin(\omega_L t). \tag{5.4}$$

This rotation affects only the x and y components of $\vec{p}(t)$, but not $p_z(t)$. The thus rotated polarization vectors obey an equation of motion in which the driving field no longer appears explicitly, but only indirectly via $R(t)$; the only field which still appears explicitly is the static part

$$\vec{h}_{\rm stat}:=\overline{\vec{h}(t)}^{t}=(\Delta_0,0,-\Delta). \tag{5.5}$$

The spin relaxation function in the GBE must be rotated, too, and is now given by

$$[R(t)\Xi(t,t')R^{-1}(t')]_{\alpha\beta} = \tfrac{1}{4}\langle[\sigma_z,\sigma_\beta];\mathcal{U}^{\Omega}(0,t')\mathcal{U}_S(t',t)\mathcal{U}^{\Omega}(t,0)[\sigma_z,\sigma_\alpha]\rangle_\lambda^{(t')}, \tag{5.6}$$

where $\mathcal{U}^{\Omega}$ denotes the spin evolution operator associated with the driving field alone, at zero static field. Since

$$[R(t)\Xi(t,t')R^{-1}(t')]_{zi}=0 \quad \forall\ i=x,y,z, \tag{5.7}$$

the z component of the polarization vector satisfies the equation of motion

$$\frac{d}{dt}\tilde{p}_z(t)=-\Delta_0[\cos F(t)\tilde{p}_y(t)+\sin F(t)\tilde{p}_x(t)]. \tag{5.8}$$

This equation, together with the corresponding (more complicated) equations for $\tilde{p}_x(t)$ and $\tilde{p}_y(t)$, has provided the basis for an earlier analysis [12] of the unbiased ($\Delta=0$), dissipationless TLS. In Ref. [12] the authors find in the *fast-driving limit* ($\omega_L \gg \Delta_0$) the driven TLS to be equivalent to a TLS without external field, but with renormalized tunneling parameter $\Delta_0 J_0(2\Omega/\omega_L)$ instead of Δ_0; and, accordingly, describe the dynamics by

$$\tilde{p}_z(t)=\cos[\Delta_0 J_0(2\Omega/\omega_L)t]. \tag{5.9}$$

The authors conclude that in the fast-driving limit and for small but nonzero $J_0(2\Omega/\omega_L)$ the dynamics exhibits strong slow mode oscillations (''low-frequency generation''); while for $J_0(2\Omega/\omega_L)=0$ the component $\tilde{p}_z(t)$ stops evolving at all: it remains forever ''frozen'' in its initial state (''dynamical localization'').

In order to understand and later generalize this result in our formalism we, too, invoke the fast-driving limit: we assume that the driving frequency is much larger than both tunneling frequency and bias,

$$\omega_L \gg \Delta_0,\Delta. \tag{5.10}$$

Yet we do not assume the TLS to be unbiased or dissipationless. The fast-driving limit allows one to replace rapidly oscillating terms by their time averages:

$$R(t)\vec{h}_{\rm stat}\to\overline{R(t)\vec{h}_{\rm stat}}^{t}=(\Delta_0 J_0(2\Omega/\omega_L),0,-\Delta)=:\vec{h}_{\rm eff}, \tag{5.11}$$

$$\langle\vec{\tilde{\sigma}}\rangle_{h(t')}\to\overline{\chi(t')R(t')\vec{h}(t')}^{t'}=:\vec{p}_{\rm as}, \tag{5.12}$$

$$[R(t)\Xi(t,t')R^{-1}(t')]_{\alpha\beta}\to\overline{[R(t)\Xi(t,t')R^{-1}(t')]}^{t'}_{\alpha\beta} =:\tilde{\Xi}_{\alpha\beta}(t-t'). \tag{5.13}$$

The latter average is taken at fixed $(t-t')$. With these replacements the GBE (3.9) acquires the convolution form

$$\dot{\tilde{p}}_\alpha(t)=[\vec{\tilde{p}}(t)\times\vec{h}_{\rm eff}]_\alpha-\int_0^t dt'\ g(t-t') \times\sum_\beta\ \tilde{\Xi}_{\alpha\beta}(t-t')\chi^{-1}(t')[\tilde{p}_\beta(t')-p_\beta^{\rm as}], \tag{5.14}$$

in particular,

$$\frac{d}{dt}\tilde{p}_z(t)=-J_0(2\Omega/\omega_L)\Delta_0\tilde{p}_y(t), \tag{5.15}$$

which predicts a slowing down of the time evolution of $\tilde{p}_z(t)$ near $J_0=0$.

To this time-averaged GBE there are rapidly oscillating correction terms which are negligible as long as $J_0\neq 0$, but which may become important at the localization transition $J_0=0$. Indeed, numerical studies by Makarov and Makri [18] indicate that at $J_0=0$ the rapidly oscillating correction terms cause additional phase relaxation (if the system is coupled to a heat bath), and eventually destroy the dynamical localization. Hence the time-averaged GBE (5.14) can only describe the dynamics near, but not exactly at, the localization transition. Furthermore, even away from the localization transition, the time-averaged GBE misses short time transients (Gaussian or algebraic decay) arising from the bath decorrelation and from the nonlinear time evolution induced by the laser field. Our description is thus expected to be valid on intermediate time scales $\omega_c^{-1},\omega_L^{-1}\ll t\ll\tau_\alpha$, where the $\{\tau_\alpha^{-1}\}$ are the rates at which the various components of the polarization vector $\vec{\tilde{p}}(t)$ relax to their stationary state.

A full analytical solution of the dynamics of the polarization vector $\vec{\tilde{p}}(t)$, and hence a study of its dynamics close to the roots of the Bessel function J_0, is only possible if we invoke further approximations: namely, the *high-temperature limit*, $T\gg\Delta_0,\Delta,\Omega$, and the *high-frequency limit*, $\omega_L\gg\omega_c$. In the high-temperature limit we may evaluate both $\Xi(t,t')$ and $\chi^{-1}(t')$ in $\rho_S[0]=\frac{1}{2}\mathbf{1}_S$ rather than in the time-dependent state $\rho_S[\vec{\lambda}(t')]$, allowing us to replace

$$\langle\ \rangle_\lambda^{(t')}\to\langle\ \rangle_0,\quad \chi^{-1}(t')\to 2/\beta. \tag{5.16}$$

The product of evolution operators in the rotated spin relaxation function (5.6) can be written as a time-ordered exponential

$$\mathcal{U}^{\Omega}(0,t')\mathcal{U}_S(t',t)\mathcal{U}^{\Omega}(t,0)$$
$$= T\ \exp\left[-\frac{i}{2}\int_{t'}^{t} dt''[R(t'')\vec{h}_{\rm stat}]\cdot\vec{\sigma}^{\times}\right], \quad (5.17)$$

where $\sigma^{\times}$ denotes the commutator with σ. In the high-frequency limit the integrand in the exponent oscillates rapidly during the interval $[t',t]$, which has a typical length $\tau_{\rm mem}\sim 1/\omega_c \gg 1/\omega_L$. In order to perform the average (5.13) we may thus replace

$$\int_{t'}^{t} dt''[R(t'')\vec{h}_{\rm stat}] \rightarrow (t-t')\overline{R(t'')\vec{h}_{\rm stat}}^{\,t''}, \qquad (5.18)$$

which will yield just the time-evolution operator of a static tunneling problem with new parameters (5.11). Defining

$$\epsilon := \sqrt{\Delta^2+\Delta_0^2 J_0^2(2\Omega/\omega_L)}, \qquad (5.19)$$

$$u := \Delta_0 J_0(2\Omega/\omega_L)/\epsilon, \qquad (5.20)$$

$$\overline{u} := \Delta/\epsilon = \sqrt{1-u^2}, \qquad (5.21)$$

the rotated spin relaxation function is then given by

$$\tilde{\Xi}(\tau)=\begin{pmatrix} \cos(\epsilon\tau) & -\overline{u}\sin(\epsilon\tau) & 0\\ \overline{u}\sin(\epsilon\tau) & u^2+\overline{u}^2\cos(\epsilon\tau) & 0\\ 0 & 0 & 0\end{pmatrix}. \qquad (5.22)$$

We recognize that now the dynamics has the same form as in the linear response regime (Sec. IV), with the additional simplification $\beta\epsilon \ll 1$. Hence, indeed, the driven system can be mapped onto a time-independent TLS with modified parameters. The results for the time evolution of $\tilde{p}_z(t)$ can immediately be taken over from Eq. (4.4), with the sole replacement

$$\Delta_0 \rightarrow \Delta_0 J_0(2\Omega/\omega_L), \qquad (5.23)$$

and consequently, $u_0\rightarrow u$, $\overline{u_0}\rightarrow\overline{u}$, and $\epsilon_0\rightarrow\epsilon$. Noting that $(\tau_\alpha^{(0)})^{-1}\propto\Delta_0^2$ [Eq. (4.5)] we find that the relaxation rates must be modified according to

$$(\tau_\alpha^{(0)})^{-1}\rightarrow[J_0(2\Omega/\omega_L)]^2\ (\tau_\alpha^{(0)})^{-1}. \qquad (5.24)$$

In the unbiased case $(\Delta=0)$ and away from the roots of the Bessel function the system thus reaches its steady state on the time scale $\tau_2^{(0)}/[J_0(2\Omega/\omega_L)]^2$. Near the localization transition $(J_0\rightarrow 0)$ this time scale diverges faster than the tunneling time $[\Delta_0 J_0(2\Omega/\omega_L)]^{-1}$, dissipation thus being effectively switched off. As one reads off from the linear response solution (4.4), this implies that there will be a slow mode coherent oscillation near the localization transition, and that the particle will remain trapped in one well on the time scale $[\Delta_0 J_0(2\Omega/\omega_L)]^{-1}$. This is consistent with Dakhnovskii's earlier analysis within the framework of the NIBA [16], and with the *ab initio* numerical calculations of Makarov and Makri [18].

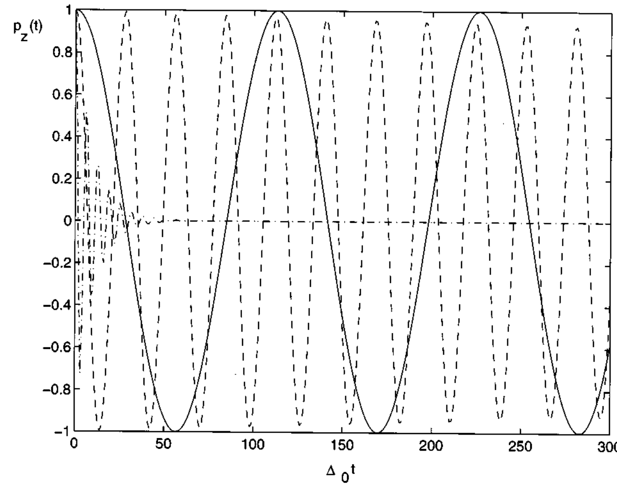

FIG. 1. Time evolution of $\tilde{p}_z(t)$ in the symmetric case. The parameters are $T=10\Delta_0$, $\Delta_0\tau_2^{(0)}=10$, and $\Delta=0$, for driving fields with $2\Omega/\omega_L=0$ (–·–), $2\Omega/\omega_L=2$ (– –), and $2\Omega/\omega_L=2.3$ (—). The localization transition occurs at $2\Omega/\omega_L\approx 2.405$.

In the biased case $(\Delta\neq 0)$, on the other hand, the amplitude of the oscillatory term in Eq. (4.4),

$$u^2=[J_0(2\Omega/\omega_L)\Delta_0]^2/\epsilon^2, \qquad (5.25)$$

becomes negligible for $J_0(2\Omega/\omega_L)\ll\Delta/\Delta_0$. One thus expects pure exponential decay, rather than slow mode oscillations, in the immediate vicinity of $J_0=0$. (One way of viewing this phenomenon is that as the bias is switched on, the quasienergy levels cease to cross and hence the destruction of tunneling can no longer be coherent.) The particle will then remain trapped in one well on the time scale $\tau_1^{(0)}/[J_0(2\Omega/\omega_L)]^2$. Further away from the localization transition the exponential decay will be superimposed with small-amplitude oscillations.

The evolution of $\tilde{p}_z(t)$ in the two cases (unbiased and biased) is illustrated in Figs. 1 and 2, respectively. We have calculated the curves using Eq. (4.4), with the replacement (5.23).

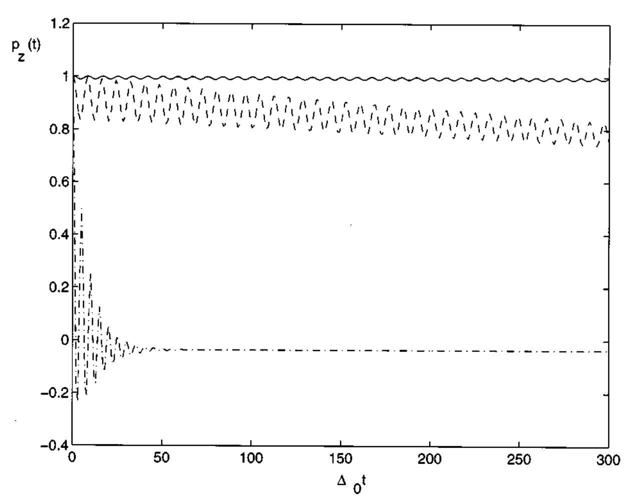

FIG. 2. Time evolution of $\tilde{p}_z(t)$ in the biased case. The parameters are chosen as in Fig. 1, except for $\Delta=0.75\Delta_0$.

VI. SUMMARY AND FINAL REMARKS

In this paper we have studied the influence of a weakly coupled thermal environment on dynamical localization. Motivated by the breakdown of the NIBA at nonzero bias and weak dissipation we have investigated the dynamics of a biased two-level tunneling system strongly driven by an external field and weakly coupled to a super-Ohmic heat bath. We have employed the Robertson projection operator formalism to derive generalized Bloch equations for the polarization vector $\vec{p}(t)$.

Assuming only the validity of the two-state picture and of the Born approximation, and invoking the fast-driving, high-frequency, and high-temperature limits, we have shown that on intermediate time scales the driven dissipative TLS can be mapped onto a dissipative TLS without driving, but with renormalized parameters. We have found that since the phase relaxation rate decreases quadratically with $J_0(2\Omega/\omega_L)$, while the tunneling frequency decreases only linearly, coherence is restored near the localization point of an unbiased TLS $(\Delta=0)$. This manifests itself in slow mode coherent oscillations near the localization transition, a result that Dakhnovskii had previously obtained within the framework of the NIBA. In the biased case $(\Delta\neq 0)$ we found a qualitatively different behavior, namely, pure exponential decay in the immediate vicinity of $J_0(2\Omega/\omega_L)=0$.

Let us finally comment on two other papers which address the problem of driven tunneling dynamics in the weak-dissipation limit [14,15]. Dittrich, Oelschläger, and Hänggi [15] consider dynamical localization in the weak-coupling limit (Born approximation) and solve numerically the quantum master equation for the full continuous bistable system with Ohmic dissipation. In contrast to our present analysis, however, they employ a restricted rotating-wave approximation and assume the bath to be Markovian, i.e., they assume that the correlation time for the boson modes is negligible compared to the characteristic time scale of the double-well dynamics. For Ohmic friction this assumption is justified if in addition to the classical time scale $\tau_{\rm mem}\sim 1/\omega_c$ also the quantum time scale of the bath $(KT)^{-1}$, with K being the Kondo parameter, is shorter than the tunneling time. But in this case the dynamics is incoherent and a weak-coupling picture inappropriate. In fact, it has been shown explicitly that the Born approximation fails for a tunneling system subject to Ohmic dissipation *and* driving, and that driving renders the dynamics intrinsically non-Markovian [13]. This is not surprising since even for super-Ohmic dissipation, where the second (quantum) time scale does not exist, non-Markovian effects are essential for the determination of the correct relaxation rates [29].

Very recently Grifoni *et al.* [14] have extended their previous treatment [13] beyond the NIBA. They derive an exact master equation, which they then use to determine the dynamics at weak coupling and fast driving. Indeed, our GBE finds a counterpart in the set of equations adjacent to their Eq. (10). Yet instead of discussing the influence of weak dissipation on the dynamics near the localization transition, they use their master equation as a starting point to investigate the modification of quantum coherence *away* from the roots of the Bessel functions, in the regime $\Omega/\omega_L\ll 1$.

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